Facing Complexes – Frege, Russell and Wittgenstein

"The complex problem is now clearer to me" (Wittgenstein)

The motto is from a letter of 26.12.12 to Russell in which Wittgenstein tells of his talks with Frege when he visited him in Jena. Clearly, the "complex problem" was a main topic of their discussions. It was obviously a topic discussed also between Wittgenstein and Russell, and the letter clearly suggests that Russell should have known what Wittgenstein meant by "the complex problem", But what was it? In the following we shall try to expose some aspects of it that concerned these philosophers and touch on basic issues in ontology and theory of meaning

A molecule, a watch, the solar system and the human brain are complex things. So are endless others. What can be clearer and simpler than that? Our Intuition as well as widespread philosophical conceptions say that not only is it clear and simple, but that these things themselves are complex, independently of how they are described or conceived. For most of them we don't have names or descriptions that identify them, and most of them, we can assume, will never be accessible to our cognition, and yet, we think, they themselves are complex. However, in the following I shall try to show that at some crucial points, as seen by the founders of analytic philosophy, things are not that simple. I shall also try to sketch some of the positions these philosophers held in trying to cope with the problem of complexity.

Part of the motivation to deal with "the problem of complexes" stems from a widespread philosophical view of truth, that when a proposition (or thought, belief etc.) like "The Earth is round" is true, there is something in the world that "makes it true". Is this a sustainable position? What is the nature of these truth-makers? Are they complex objects? Or perhaps complexes of another kind? We shall see that our three heroes had different views about these questions. And the issues are not easy to trace – both the views, the arguments for them and against the alternatives. Though my ostensive purpose here is to provide a (partial) interpreted survey of these positions, I shall also try to briefly explain some of the relative merits and deficiencies of each as seen by the others.

The distinction between simple and complex is basic in philosophy since its beginnings. It has great importance, for example, in the philosophy of Plato (a famous example is the "Dream Theory" in Theaetetus 202d-210), and has remained, in this or that form, a major philosophical concern ever since. Part of its significance is that complexity

is a necessary feature of a notion of analysis: According to a common view, in many analyses, we decompose complexes to their simple (or less complex) components.

However, assuming for the moment we have some conception of a complex, does the world contain complexes? Are complexes real, over and above their constituents? Are they kind of objects? Is the complexity of something an objective feature of it or an aspect of our ways of describing it or referring to it? These and similar problems are more pertinent to some philosophical views and less to others, but they were central interests of our three founders of analytic philosophy, to some of their views we now turn.

Frege

It is Frege's theory of meaning[1] that sets the stage for the complex problem we are discussing. I shall argue in the sequel that the notion of complexity, which is at the basis of the possibility of analysis, is one of the sources of Frege's distinction between sense and reference, and is at the basis of his position that a theory of meaning must be bi-dimensional and concern sense as well as reference. This is not the standard way in which this celebrated distinction is presented, and it is not the way Frege explicitly introduced it (mainly in SR and BL)[2]. But it is implied, as I shall argue, by basic features of his position. One is his conception of reference; another is his conception of ontology. I begin with a brief overview of these, focusing on the primacy of language in his conception of ontology, and on his view that objects are the references of names, and functions (including concepts and relations) – of predicates, and that they differ categorically from each other. These themes are in general fairly well known, and I shall be brief and sparse in textual references. I shall then suggest some intriguing implications of these views to the issue of complexes, which were not stated explicitly by Frege. Although there are exegetical problems and disputes on some of the following points, I believe they adequately represent basic trends in Frege's thought, but defending it in detail will take us too far afield.

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[1]   I shall use "meaning" as a general term, neutral as between sense (*Sinn*) and reference (*Bedeutung*).

[2]   Though the systematic distinction between reference and sense and the use of this terminology begins in the papers of 1891-2, there are anticipations of the idea in *Begriffsschrift* (especially §8) and *Grundlagen* (FA) . See Bar-Elli (2006),

Frege's Linguistic Revolution in Ontology – One of the revolutionary insights of Frege's work is that ontology and ontological categories are conceived in terms of a theory of meaning. The main steps in the argument for this can be presented as follows: Ontology is the theory of the nature of existence and its basic categories. The notion of existence and its meaning is determined by the meanings of statements of existence. A prime task in clarifying these meanings is determining their logical form, and explicating the meanings of their constituents – what they are about. Here in three steps we are at the heart of Frege's theory of meaning: The dependence of the meaning of a term on that of statements containing it ("the context principle"), and clarifying this by exposing their logical form, which determines *inter alia* their implication relations, and the things referred to by their referring constituents, which are the things statements containing them are about.

Such a theory is built, according to Frege, on his logical language – *Begriffsschrift* (conceptual notation) – and on his theory of reference (*Bedeutung*) and sense (*Sinn*). I cannot delve here into the details of this theory, but will mention in a very general way that it concerns determining a reference for any expression in logically analyzed propositions, where the reference of an expression is what propositions containing it are about and what determines their truth or falsity.[3] This theory is conceived within a general doctrine that the reference of a term is always given to us in a particular way – its sense – and senses build up thoughts, which are senses of (indicative) propositions and are essentially true or false. On a more specific level it implies that the reference of a name (proper or general) is an object (concrete or abstract), and the references of predicates are functions.[4] Concepts and relations are functions, where some are from objects to truth values, and others (second-order) are from the first-order ones to truth values. Existence is such a second-order concept that applies to (first-order) concepts. Thus, saying that there are Fs, or that Fs exist amounts to saying something of the

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[3]   For the basic role of the notion of about in Frege's theory see Bar-Elli (1996), ch. 7.

[4]   These ideas, present already in FA, are systematically introduced in FC, SR and CSR.

concept F, i.e. that it is instanced, or that some objects fall under it, or that it is true of some objects.[5]

Saying that the reference of a proper name is an object does not only mean that the general notion of an object is conceived as linked to the general notion of the reference of a proper name; it also means that our notions of specific objects, conceiving them as objects, are thus linked to conceiving them as the references of specific proper names. Now, in saying that, are we stating what an object is or what is a proper name, or perhaps both? Frege is not clear on that, and this is a point of dispute among his scholars. On a first look it seems that the first possibility is the correct one. Indeed this is how Frege's position is often presented: we are supposed to have prior independent notions of object and of specific objects, and we explicate the technical notion of reference in their terms. According to a bolder view (propounded e.g. by Dummett[6]) it is the second that expresses Frege's deeper view: We have no notion of an object, according to this view, save as the reference of a proper name.[7] It is the theory of reference plus a prior

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[5]   In FA Frege says "Existence is a property of concepts" (FA, 103/65, in FR). The idea was emphasized repeatedly since then (see e.g. letter to Liebmann of 25.08.1900, PMC, 93). This is a central thesis in FA, e,g, §§ 47-50. In the Introduction to BL (5) it is claimed to be the main discovery of FA. It is worth noting that in the early "Dialogue with Puenjer on Existence" (PW, 53-67/60-75) Frege held a different view, according to which existence is part of the logical form of the appropriate proposition, somewhat, as he says, like a copula.

[6]   See Dummett, M.: *Frege, Philosophy of Language*, Duckworth, 1981 ch. 4. I shall not repeat here the forceful arguments and references Dummett brought for this view. Let me just emphasize that for Frege the notions of object and function are correlative, as he makes clear e.g. in FC, where he writes regarding the former: "We have here something too simple to admit of logical analysis ... **an object is anything that is not a function, so that an expression for it does not contain any empty place.**" (FC 140/18 in FR)

[7]   For instance, in CSR in explaining the reference (*Bedeutung*, "meaning" in PW) of function-words Frege writes about their values: "I call such a meaning (*Bedeutung*) an object..." (PW 119/129). This seems to explain the notion of object in terms of the reference of a name.

conception of the linguistic category of proper names that constitute our notion of an object, i.e. of what we grasp in regarding something as an object. Likewise for predicates and functions: Our notion of a function (including concepts and relations) is essentially linked to conceiving functions as references of predicates or function-expressions. One aspect of what we thus grasp is that the notions of object and concept (function) are correlative: there is no understanding of the one without the other.

Without delving here into the issue, I incline to think that the third option is more faithful to Frege's thought. "Object" "function" "concept" "reference", "existence" etc. are extra-theoretical terms that do not designate concepts that can be expressed within the logical language (*Begriffsschrift*). And yet they belong to the "conceptual envelop" that surrounds the theory and in whose framework it is introduced and gets its significance. The notion of reference is conceived and explained by theoretical principles governing it, in whose basis is the elementary and indefinable notion of truth. But apprehending it cannot be detached from some conception of the notions of object and concept. Granting some prior conception, perhaps vague and unclear, of these, regarding objects as the references of names is a reasonable consequence of these principles. These two pillars – the theoretical principles of reference and the intuitive understanding – support and enhance each other. Almost whatever we say here belongs to this ambient framework.

We can somewhat crudely summarize the above by the following principle, which admittedly was never stated by Frege, but is behind much of his work:

A) The meanings of expressions of our language are constitutive of our conception of things in the world – objects (including complexes), concepts, facts, etc.

Concepts and functions in general are, as said above, references of predicates. They differ from objects categorically, in that by their very nature they are "incomplete", "unsaturated" (*ungesaettigt*).[8] Frege's metaphorical formulations here are not entirely clear, and rely heavily on the incompleteness of predicates. In general the idea is that unlike objects that are "self-subsistent", concepts are essentially conceived as applying to objects or as being true (or false) of them. From this perspective they are unsaturated in themselves. This incompleteness is expressed by the empty places in the linguistic

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[8]   Frege was never tired of emphasizing this. See for example, FC (133-4/7 in FR) and his letter to Marty of 29 August 1882, (PMC 101; FR 81).

expressions designating them (here by the three dots; in logic by the use of variables), which are essentially predicative. The entire content, the essence of the concepts referred to by "...is even", "...is wise", "... is bigger than ---" etc. is in their applying to and being true of some objects and false of others. Devoid of this they are meaningless, they are not "self-existent".[9] This does not deprive functions and concepts of their objectivity; they are as real and objective as objects. Neither does it exclude empty concepts – concepts that in fact do not apply to anything. Yet they are unsaturated, in the sense that their possible applicability to objects is essential to them. The elementary notion of truth and the primary role of complete propositions are at the basis of this conception: An object falls under a concept when inserting the object's name in the empty place of the concept-expression (the predicate) forms a true proposition.

Unlike concepts, objects, according to Frege, are "self-subsistent" (*selbstaendig*). Their nature and existence do not have this essential dependence on other things. Objects, of course, have properties and satisfy some concepts, and relations, but on Frege's view this is not essential to them in the way applying to objects is essential to concepts. This position of Frege's has been widely discussed and often attacked, and I will not try to defend it here.

There seems to be a tension between this characterizations of objects and concepts, which seems directly ontological, and principle A. If there is, it is internal to Frege's thought, but I believe that it can be ultimately resolved: the "ontological" notions of incompleteness and self-subsistence remain unclear unless linked to the linguistic and semantical notions of proper name, predicate, sentence, satisfaction and truth. I cannot indulge here into a more detailed discussion of this intricate issue, and it is not necessary for our argument here.

Yet, it is important of realize the significance of the above ontological conception to our concerns. We said at the beginning that according to a widespread position, things themselves (a molecule, the brain) are complexes, independently of the ways they are conceived or described. In the light of the above Fregean conception this is a problematic or confused view. On the above bold interpretation, we have no conception of an object save as the reference of a name (including a possible one that is not actually in use), and no conception of a function save as the reference of a predicative expression. Of course, this does not imply the absurd view that there were no objects or functions prior to

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9   Compare Russell's explanation of his somewhat similar distinction in PLA 205.

language. In saying that the notions of names and reference are constitutive of our notion of object we are not talking about existence or existential priority. We are talking rather about what is conceptually involved in regarding something as an object. This, according to the bold conception, is basically regarding it as a possible reference of a name. Likewise. for the notion of function: regarding something as a concept or a function is regarding it as a possible reference of a predicate or an appropriate expression.[10]

For the present concerns the difference between the bold interpretation and the more flexible one we favored is not crucial. Even on the latter the notions of reference and our conception of ontological categories cannot be detached from each other, so that there is no sense in talking of the things "themselves" independently of their being such (possible) references. By this Frege gives a sort of philosophical explication to the common notions of object and concept. He presumes some partial and blurred conception of them in their common use, and tightens it up and makes it more precise in the framework of his theory of reference. We shall see in the sequel some aspects of the significance of this to our concerns.

We can return from here to our main topic – complexes. A complex is constituted by having definite parts and structure. A principle much less discussed than (A), is that in general,[11] on Frege's theory of reference, **the reference of a part** (of a complex expression) **is no part of the reference of the whole**. Taking, for instance, "the capital of

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[10] In *Begriffsschrift* §9 a function is defined as a certain linguistic expression (some regard it as a sloppy formulation due to use/mention confusion). This is explicitly rejected in FC (131/3-132/4 in FR), where function is clearly the *Bedeutung* of an appropriate expression. But even there we have no explanation of what a function is save as the reference of a functional expression, which is introduced as what is common to various expressions, e.g. $2 \cdot 1^3 + 1$, $2 \cdot 2^3 + 2$, $2 \cdot 3^3 + 3$ etc.. Likewise, in explaining his extension of the mathematical notion to non-mathematical expressions Frege says: "We split up the sentence 'Caesar conquered Gaul' into 'Caesar' and 'Conquered Gaul'. The second part is unsaturated ... Here too I give the name 'function' to the *Bedeutung* of this unsaturated part." (139/17 in FR).

[11] "In general" because there are exceptions, as when something is identified by parts it contains, e.g. "the county to which Cambridge belongs".

Sweden" as a complex name referring to Stockholm, Sweden is no part of the capital of Sweden, nor are the references of numerous other terms in expressions referring to it; Likewise, 5, 3 and the minus function are no parts of the number 2, which is the reference of "5-3"; similarly for concepts and functions in general, as can be seen by considering several definitions of a concept, e.g. a circle.[12]

These examples and endless others exemplify the following principle:

> **B**) The reference of a part (of an expression) is in general no part of the
> reference of the whole.

This means that in considering the contribution of the reference of a whole expression to determining the truth-value of propositions and to understanding them, we cannot talk of the constituent parts of the reference, of its structure and complexity. Since complexity consists in the constituent parts being determinate, this means that there is no sense in talking of a complex reference (object or function). This is no small matter, for it is crucial for understanding how different expressions with different constituent parts can refer to the same thing (object, or function), which is vital for science, mathematics and thought in general.[13] Disregarding that principle we might think that the references of the parts of "5-3" are parts of the reference of the whole, and this would rule out its identification with the reference of e.g. "8:4", which is absurd. For similar reasons, when concepts are strictly understood in a Fregean way as references, they also can't be analyzed. Somewhat provocatively, we can say that strictly, on Frege's view, there is no conceptual analysis.

An important corollary is that although the references of the parts determine that of the whole, this does not hold in the opposite direction: the reference of a whole expression and that of a part do not determine the references of the other parts.[14] This is

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[12] See Frege's "Notes for Ludwig Darmstaedter" (1919), PW 255, (FR 364-5), though there are some unfortunate formulations in earlier writings. Dummett rightly remarks that the idea that the reference of a part is part of the reference of the whole "manifestly absurd" (1981, 159; 1981b, 65).

[13] This central strand in Frege found an emphatic conclusion in Wittgenstein's doctrine in the *Tractatus* that "Objects are simple" (2.02). We shall return to it in the sequel.

[14] This is stated in an unhappy formulation already in SR (150/36 in FR) when Frege writes "The whole *Bedeutung* and one part of it do not suffice to determine the

evident with regular mathematical functions: 3 as argument for $(x)^2$ determine 9 (as value), but 3 and 9 do not determine a function (there are endless functions to which they belong, e.g. x+6). On Frege's theory of reference this is true also of complete propositions: The references of "a" and of "F" determine that of "a is F", while the references of "a" and of "a is F" do not determine the reference of "F".

This, let me add, shows that the above does not conflict with Frege's famous principle of compositionality, according to which the reference of a complex expression is determined by the references of its constituents – is a function of them. Sure, the reference of "the capital of Sweden" is a function of the references of "Sweden" and of "the capital of---". But this does not mean that they are parts or constituents of it. Similarly it is evident that 2 is determined as the reference of "5-3", but it is absurd to think that 5 and 3 are parts of 2. It should also be remarked that the above principle about complexity is different from Frege's celebrated context principle, even if that principle holds also of reference (which some people doubt).[15]

All this is of course entirely different with regard to sense (*Sinn*): In general, the sense of a part is a part of the sense of the whole. The sense of "the capital of Sweden" does contain the senses of "Sweden" and of "the capital of---". It is not only a function of them, but built up by them. Likewise for senses of sentences like "5-3<4". Senses have structure and determinate constituents (though there may be exceptions to this). They can be complex in a strict sense. What we usually call conceptual analysis is therefore not an analysis of a concept but of its sense, or of the concept under its sense.[16] Though, for simplicity, I speak here of concepts, their referring predicates and their senses, much of what I say here applies to other kinds of reference (objects, functions) and to their senses, in particular thoughts, which are senses of propositions and are often what is analyzed. Since analysis is a fundamental cognitive task, the above may be seen as a major source

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remainder", where his intention is clearly as stated in the text, namely that the *Bedeutung* of the whole (expression) and that of one part of it do not suffice.... This is evinced also by his own explanation in the same paragraph that parts and analysis are discerned in the sense, not in the reference by itself.

[15]   On the relationships between the context principle and compositionality see Bar-Elli (1996) ch. 5.

[16]   For a detailed discussion of this see Bar-Elli (2017).

of the Fregean distinction between reference and sense and of the need to include the latter in a theory of meaning.[17] We thus get the crucial principle:

C) Complexity, analysis, determinate structure and constituents belong primarily to senses. Derivatively, when applied to references this is done only via their senses or when they are conceived under their senses.

Since these notions are of prime importance in logic, philosophy, mathematics and almost any field, this shows the need to recognize sense in addition to reference and that a theory of meaning for these fields must include also the notion of sense.

We started by saying that obviously something like the solar system is complex. Call it S. The solar system is then the reference of "S" and it is obviously a complex whose parts are the sun, the planets etc. Here we have a complex which is the reference of S. How could Frege deny it? He did not. There are two mistakes in thinking that he did. One is in thinking that a complex which is the reference of an expression is a complex reference of that expression. The latter, unlike the former, means that as a reference it is complex, i.e. that its complexity and structure function in determining the truth values of propositions about it. This, as we have seen is in general not the case. But this does not mean that when we conceive of something (like S) as complex it cannot be a reference of a term. The second mistake is in ignoring that we conceive S as complex in conceiving it under the sense of "the solar system". In fact a reference is never conceived "in itself" but is always conceived in a certain way, under a certain sense: "It is via a sense, and only via a sense, that a proper name is related to an object" (CSR, PW 124/135, FR 180). And many things – alleged complexes – are conceived in terms of their parts. A molecule is often conceived as a structure of atoms; the solar system – as a structure of planets revolving around the sun. This is a feature of their senses, which is at the basis of their alleged complexity.

This does not mean that the references themselves are not concerned. Note that in explaining the second mistake in the above paragraph, I said that in thinking of the solar system as complex we conceive **it** under a sense. This is also why in principle (C) we said that complexity is a scribed "primarily" to sense. We don't exclude complexity from things that are references. But when we conceive them as complex we do so in virtue of some features of those senses under which they are conceived. These features are specific to each case. It is not, to prevent another misunderstanding, the mere complexity of the

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[17]   For a fuller account of some features see Bar-Elli (2010), (2015).

sense that is "transferred" to the reference: " the point of intersections of the medians in a triangle" has a complex sense and a simple reference. But conceiving something, which is the reference of an expression, as complex, is due to some features of the sense under which it is conceived.

One should give due weight here to the internal relationship between sense and reference. Sense, though real and objective, is a way in which reference is conceived or given. It is not a separate and independent entity that stands in some contingent relation to its reference. Sense is strongly supervenient on reference and stands in internal relation to it. Talk of a sense of an expression is therefore also talk of its reference.[18] Hence, in stressing, as we did above, that on Frege's view in saying that something is complex we conceive it under a particular sense does not mean that we are talking of something else – we conceive **it** under a particular sense.

Frege regarded many complex expressions (like definite descriptions) as proper names. Since many such expressions seem to lack reference this posed an acute problem for him for, by basic principles of his view, he could not regard them as having sense but lacking reference. This is a notorious problem, both substantive and exegetical, which was widely discussed. I shall not get into it here (See e.g. ***). Let me just mention that I side with those who think that in spite of some formulations about natural language (mainly in SR), by basic trends of Frege's philosophy there are no referenceless senses: A thought, for Frege is essentially true or false. It can be either only if all its constituents have reference. Hence, in genuine propositions that express genuine thoughts any expression having a sense has a reference[19]. This is also in accord with his conceiving of sense as a mode of presentation (*Art des Gegebensein*s) of a reference, which hardly makes sense when there is no reference. In talking of referenceless expressions in a natural language (mainly in fiction, poetry etc.) Frege was careful to remark that these expressions are useless in science and do not express genuine senses and thoughts.

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[18]   I expanded on it and on the way it solves some puzzles of Frege's theory of oblique contexts in Bar-Elli (1996) chapter. 9.

[19]   This is a firm conviction of Frege's since what Dummett considers his earliest philosophical piece we have – '17 Key Sentences on Logic' (no. 10), PW, 175/189; It is asserted again in e.g. FC of 1891, 141/19-20 in FR; "On Schoenflies" of 1906 in PW 180/194.and numerous other places.

Indeed, in his formal language he devised a way of allotting e.g. definite descriptions references so that whenever such an expression seems to lack reference, a proposition containing it turns out false (and not truth-valueless).

Complexity, structure and analysis apply in Frege not to the references in themselves but to their senses, or to conceiving them under their senses. How then would see it those who reject Frege's notion of sense or its inclusion in the theory of meaning? This was the position of both Russell and Wittgenstein, to whose views we now turn, with a special focus on the alleged complexity of facts, and of what is described by a complex definite description. Their positions on the issues were heavily influenced and triggered by Frege, sometimes by severe criticism of his views. This is truer of Wittgenstein than of Russell, who was quite alien to the above Fregean revolution in ontology and to the primacy of language (principle A). He was therefore less troubled by the above principle (B), for he thought he could conceive complexes directly, apart from their being referents or designations of linguistic phrases. Wittgenstein, on the other hand, accepted the primacy of language (principle A) even more forcefully than Frege, but objected to other basic features of Frege's theory, some of which will be mentioned below. We shall not examine these critiques, and rather concentrate on their own conceptions of complexes.

Russell

In spite of many changes in his philosophical views, Russell thought most of his long career that facts (and propositions) are complexes. In the period on which we shall focus (1903-1918) he thought moreover, that complexes are object-like, and that facts are essentially complex objects, like "a standing in the relation R to b".[20] In *Principles of Mathematics* of 1903 (PoM) he wrote: "For example, 'A differs from B' or 'A's difference from B,' is a complex of which the parts are A and B and difference' (p. 139). The first is a proposition, the second is an object, and Russell runs them together as a complex.

In *Principia Mathematica* of 1910 (PM), e.g. he (and Whitehead) wrote: "We will give the name of 'a complex' to any such object as '*a* in the relation *R* to *b*' or '*a* having

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[20] Russell may have got the general idea of a complex and its importance from G.E. Moore. The idea that a proposition is a complex of objective concepts that stand in external relations was propounded by Moore in his 'The Nature of Judgment' (*Mind*, 1898) and 'The Refutation of Idealism' (*Mind* 1903). Russell was much influenced by this (see Hylton (1990) ch. 4; Potter (2009), 12-13).

the quality *q*,' or '*a* and *b* and *c* standing in relation *S*.' Broadly speaking, a complex is anything which occurs in the universe and is not simple" (44). The ending leaves room for many possibilities, and Russell later (e.g. PLA of 1918) posited kinds of complex corresponding to negative, molecular, general, and existential propositions.

With the exception of the OD theory of 1905, of which we shall talk later, Russell talked abundantly of complex objects and concepts, and did not clearly distinguish between the kind of complexity of a proposition (and of a fact), and that of an object or a concept. All these are "things occurring in the universe which are not simple" to use the above quoted formulation.

In general, complexes for Russell are built up by their constituent parts and have determinate constituents and structure. How this is related to the meaning of phrases denoting or meaning them is, as we shall see, a central problem for which Russell may not have had a consistent answer. There are three main kinds of complex Russell dealt with: a) complex objects like a watch, and b) complex concepts like equilateral red triangle, and c) facts, in particular atomic facts like that Cain killed Abel, which Russell didn't really distinguish from the complex of Cain killing Abel. One problem with complex objects is that they may not exist even when all their constituents do. Thus if a watch breaks and all its parts are spread around, it does not exist even though all its constituent parts do. What then is it that constitutes the complex over and above its constituents? This and other problems pertaining to complex objects of type (a) are treated by Russell's famous theory of descriptions (originating in his OD).

According to this theory propositions allegedly about such objects (when the description is the main operator) are analyzed as existential, so that if the object does not exist, they turn out false (more on it later). This theory also deals with special complex concepts Russell earlier called "denoting concepts" (PoM ch. 5). These are concepts whose occurrence in a proposition makes it be not about them (as with other concepts) but about something else connected to them "in some peculiar way". For example, the point of intersection of the medians of a right-angled triangle is a complex concept denoting a simple point.[21] But how is this denoting effected, and what is the nature of this "peculiar way"? In the unpublished "On Meaning and Denotation" of 1904 Russell is troubled by this problem and by the fact that the meanings of the parts of a complex expression are no parts of its denotation: "...the whole phrase designates something of

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21   *The Principles of Mathematics*, 1903 (PoM), ch. 5, p. 53.

which the designations of the parts are no parts ... England is not part of Mr. Balfour [the designation of 'the prime minister of England']" (quoted in Hylton 1990, 257). He considers it as motivating the need to distinguish the (complex) meaning of a complex expression from its denotation. Thus, without endorsing Frege's notion of reference, Russell recognizes something like the above principle (B). This poses a problem as to how meanings contribute to determining this denotation, which is not really answered there. We shall explain below why Russell was not as troubled by it as Frege was.

Indeed, the OD theory of 1905, published just few months later, dispensed with such complexes, and provided an analysis of the relevant propositions, in which these phrases (mainly definite and indefinite descriptions) are dubbed "incomplete symbols", which don't mean or designate anything. Thus, "the F is G" is analyzed as "There is one and only one F and every F is G", where "the F" is not a constituent and does not designate anything.[22] Part of the significance of the change is aptly put by Hylton in saying that by the OD theory the only complexity is propositional complexity. However, it is hard to say that after OD Russell consistently adhered to that insight.[23] For, by about PM of 1910 Russell abandoned his previous belief in propositions as metaphysical entities, and confined "proposition" for a linguistic sentence (many claim that he was inconsistent in that, both before and after PM). The metaphysical truth-makers are now complexes or facts, which, as we saw above, in PM were considered a kind of objects. This is in a way opposed to the above OD insight that complexity is essentially propositional. But it seems that this, that would be regarded as intolerable inconsistency by a Frege or a Wittgenstein, did not much trouble Russell. He not only rejected Frege's notion of sense, and hence, the above principle (C), but what is perhaps even more important for our concerns, he did not share Frege's revolutionary conviction that the ontological categories are constituted by the language used in talking about things and by its theory of meaning (the above principle A). He rather thought that we have a direct grasp of them, and that language is at best a means of describing it. By rejecting principle (A) Russell could regard (B) as at best reflecting an inner feature of a theory of reference or of meaning, devoid of any metaphysical significance.

The above problem of non-existence pertains also to complexes of kind (c) – how we can account for their non-existence (when the corresponding proposition is false) even

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[22] For a detailed exposition of it and its significance, see Bar-Elli (1980); and (1997).

[23] See Hylton (1990), ch. 6, particularly pp. 256-264.

though all their constituents exist – but has been widely treated. I wish to focus here on another problem which has been dealt with much less, but is important for understanding the issue between our three heroes. It has to do with the very nature and identity of a complex. Russell often implies that e.g. the complex of 2 being smaller than 3 is different from that of 3 being bigger than 2, because different relations are involved and the complexes are differently ordered.[24] But is the latter also different from $\sqrt{9}$ being bigger than 4:2? Is the complex of Cain killing Abel different from that of Adam's eldest son killing Adam's second son? If a complex is identified by its constituents, the answer should turn on what these constituents are. On the one hand it seems that the constituent is the object itself, that Cain and the eldest son of Adam is one and the same constituent. In a letter to Frege of 12.12.1904 Russell insisted that Mont Blanc, the mountain itself, is a constituent of the proposition asserted by "Mont Blanc is more than 4000 meters high". He adds: "We assert the object of the thought and this is, to my mind, a certain complex (an objective proposition, one might say), in which Mont Blanc is itself a component part".[25] On that view then, Mont Blanc and the highest mountain in Europe are the same constituent; Cain, Adam's eldest son and the first human murderer are the same constituent, and the answer to the above question must be No.

On the other hand, by Russell' theory of descriptions when the description "the eldest son of Adam" occurs in a proposition, Cain is not among its constituents, which include the components of the description (say, Adam, x being a son of y, x being older than y, etc.). The answer to the above question must then be Yes – the complexes concerned are different. A proposition about Adam's eldest son and one about the first human murderer have different constituents, definitely if constituents are governed by Russell's principle

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[24]   In *Theory of Knowledge* Russell tries to overcome this by appealing to "position in a complex", but admits that "earlier in a complex" and "later in a complex" are different constituents that are "indefinable and must be simply perceived" (145). Moreover there are many "opposites" (like easier and harder) for which there is no natural sequence as for "precedes" or "<".

[25]   PMC 169.

of acquaintance, as he repeatedly emphasized.[26] The result would be a sort of intensional view of complexes, where (roughly) the identity of a complex is sensitive to the way it is described. Neither this nor the previous answer is explicitly stated by Russell, and it is doubtful whether he had a consistent view on this.[27] It seems that his empiristic heritage presses towards the latter view, while his realistic inclinations towards the former. This is a feature of the above mentioned point that Russell thought that complexes and their constituents are known, irrespective of their being meanings of linguistic phrases.

This is a basic conviction on which Russell is at odds not only with Frege but also (and perhaps mainly) with Wittgenstein. In an introductory note to his *The Philosophy of Logical Atomism* of 1918 (PLA in LK) Russell writes that they reflect Wittgenstein's influence on him.[28] The extent of this influence and its relations to his previous views is an intricate and controversial issue, into which I cannot get here. I therefore confine this survey to some of his works prior to 1918. In *Problems of Philosophy* of 1912 (PP) Russell writes: "When a belief is true, we said, there is a corresponding fact, in which the several objects of the belief form a single complex" (78). He then says that we can perceive this complex, and thereby have "knowledge of a thing" (79), which is "acquaintance with the complex fact itself" (ibid.). "You can ... look to the west and actually see the setting sun" (ibid.), and this, for him at that time, is no different than seeing that the sun is setting. Much like in PM, complex and fact are thus compounded, and both are treated as objects, things with which we may have acquaintance.

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[26] The principle is stated already in OD (in LK p. 56), and still held in *My Philosophical Development*, p.169. In addition to the articles in note 17, I elaborated on it in Bar-Elli (1980b), pp.93-101.

[27] See on this Potter (2009), p. 103. In explaining the advantage of facts over complexes Potter writes: "But complexes do not speak: they are what they are, and do not present any particular facts as salient. Complexes, therefore, cannot be what ground propositional thought, because they do not have the right kind of structure to do so" (104). This, though in general true, seems to me a bit unfair to Russell's "quasi-intensional" conception of the constituents and structure of a complex, but may bring out its problems.

[28] Though he had not read the *Tractatus* then, see Potter (2009), p. 108.

This was one of the main features of Russell's view on facts, to which Wittgenstein explicitly opposed, and he may have persuaded Russell that it was untenable. Russell's coming to terms with the distinction between facts and complexes (construed as things or objects) matured rather slowly – undoubtedly under Wittgenstein's influence. In his correspondence with Frege in 1902-1904, Frege repeated in detail some of his reasons for the need to distinguish sense from reference.[29] Of special importance to our concerns with complexes is his detailed explanation in the letter of 28.7.1902 (PMC 139-142) of the difference between "a whole", in which a part of its part is part of the whole, and a class in which this does not hold. Technically, this related to Russell's notorious efforts to distinguish "class as many" from "class as one", but Frege was making a general point as evinced by his saying that "we regard every physical body as a whole, or system, consisting of parts" (ibid. under (2)). Frege was not talking there of facts, but he used just the same kind of considerations of the part-whole relation almost 20 years later in criticizing Wittgenstein's conception of facts as *sachverhalten* (more on this later). Russell was not persuaded. His reasons there, based in fact on his rejecting Frege's notion of sense, are not easy to trace, and I shall not endeavor it here.

In *Theory of Knowledge* of 1913 complexes still play a very large role. Judgment is now the main bearer of truth. Correspondence to an existing complex is what makes a belief or judgment true, And again, a complex is not really distinguished from a fact: "It may be questioned whether a complex is or is not the same as a fact ... there is certainly a one-one correspondence of complexes and facts, and for our present purposes we shall assume that they are identical" (79-80). One of the constituents of such complex-facts is a relation relating other terms, e.g. precedes, in A precedes B, or being smaller than in 2<3.[30] However, as we saw above, a basic trend in Russell's thought was to define a complex and its identity in terms of its components. This posed a difficulty for this conception of a "relating relation". Johnston aptly puts it by saying: "What we have with Russell here is the idea that a relation is a brick doing duty as cement between other

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[29]   See in particular the letter of 28 December 1902, PMC 152-154, and of 21 May 1903 (156-8).

[30]   This was a central idea of Russell's over a long period. In *Problem of Philosophy* of 1912 he wrote: "Whenever a relation holds between two or more terms, it unites the terms into a complex whole" (p. 74). The idea is already central in PoM (e.g. 52).

bricks" (Johnston, 234)[31]. Russell admitted it to be "a grave logical difficulty" (see PoM 466-7). Moreover, it is very unlikely that he held the objects of a complex to be also intensional in the above sense, which would make the identity of objects and of complexes quite mysterious. Russell clearly assumed that the same object can be given differently e.g. by different descriptions, or by different mental faculties like sensation and imagination, or different attitudes like belief and doubt (TK, 54), and this should now apply to the relating relation as well. Whether facts, supposedly undistinguished from complexes, are also thus insensitive to the ways their constituents are described remains unclear there.

However, since 1914 the distinction between things (including complexes) and facts became clearer in his writings. In the Harvard lectures 'Epistemology' of 1914 Russell wrote, probably under Wittgenstein's influence: "What we analyze is a fact, not a thing. A complete description of the real world is not given by enumerating the things in it" (quoted in Potter 105). In the Lowell lectures of 1914 (published as *Our Knowledge of the External World*, 1928, Mentor Books 1960, OKEW) Russell writes: "When I speak of a fact I do not mean one of the simple things in the world; I mean that a certain thing has a certain quality..." (47). He is still unclear on whether facts are things, though not simple. In the 1918 PLA the distinction between facts and complex objects begins to take a systematic hold (1st lecture). Though Russell speaks there a lot of the complexity of facts, it is not the one of objects, but of propositions (191-3) We shall not discuss it here.

It is well known that in 1912-13 Wittgenstein had harsh criticism on many of Russell's views, among them his most fundamental ones. The existing evidence relates mainly to Russell's conception of logic and the logical constants and to his theory of judgment (knowledge, belief etc.), on both of which Russell admitted Wittgenstein's critique to have profound influence on him to the point of "paralyzing" him and making him leave these subjects[32]. As far as I know Wittgenstein's critique was not directed explicitly to Russell's conception of complexes and of facts as complexes, but it is pertinent I think to these topics as well, and the fundamental nature of the issues makes this unsurprising. We shall touch on some points of the connection below.

Before that, let me remark that there is something common to Russell's and Wittgenstein' views, which makes Wittgenstein's critique even more pertinent and

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[31] Johnston (2007), pp. 231-251.

[32] This occurs mainly in his letters to Lady Ottoline. See also e.g. Blackwell (1981).

interesting. It is something they both thought was missing in Frege's theory concerning what we may call the ontological grounding of truth. We have seen that for Russell this is one of the primary roles of complexes (which he did not clearly distinguish from facts): when a belief or judgment is true, there is something in the world that is its "truth maker", a complex that "makes" it true, its ontological grounding. In Wittgenstein's *Tractatus* this is fulfilled by facts or existing states of affairs (more on this below). **There is nothing corresponding to that in Frege's theory**. Facts in his theory are senses (*Sinne*), true thoughts. They definitely can't fulfill this role; neither can objects or functions. The closest one can get to this in Frege is his talk of the "falling of objects under concepts" (functions). But this is still not a truth-maker. That 'Fa' is true and that a falls under F come to the same thing in Frege, but the latter does not ground or explain the former; such fallings under concepts are not entities in Frege's ontology; however, complexes and facts are, in Russell's and Wittgenstein's.[33] This, I believe, is the deep reason why Frege was troubled by Wittgenstein's notion of *Sachverhalt* in his letter to him of 2 June 1919, which will be discussed below. It is not just that Wittgenstein's formulations in introducing it were unclear; it is, Frege thought, that there is no such thing, and Frege's ontology does not leave room for it.

Until 1906 Russell held that a judgment (belief, and other so called "propositional attitudes") is a dyadic relation between a subject and a complex (proposition). Since 1906 to at least 1913 he replaced it by various versions of what he called "the multiple relation theory", according to which the subject stands in a multiple relation to the various objects of his judgment, or to a complex including a relation in which objects stand in certain ways. Another thing such a complex must include is what Russell called in 1913 "form" (and earlier "sense"). Thus $J(S,F,R, x_1...x_n)$ is the proposition that a subject (S) judges (J) a complex with form (F) of the things $x_1...x_n$ standing in the relation R. Moreover, according to the principle of acquaintance we must be acquainted with all these in order

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[33]   In some unhappy formulations Frege talks of the falling of an object under a concept as a relation (CSR, in PW 120/130). But this I think is meant in the special sense that the predicative copula is said to be a relation. P. Geach remarked that the notion of facts as existing entities is a late 19-century invention, *Logic Matters*, University of California Press, 1980, p. 38.

to understand the proposition. This theory was abandoned by Russell under Wittgenstein's criticism[34] and was officially discarded in the 1918 lectures.[35]

Wittgenstein objected to ascribing meaning to logical constants and to forms of propositions, not to mention their objectual ontological status.[36] This is one of the basic oppositions of his to both Frege and Russell, so that on his view J, F, R in the above formula cannot be designating terms and do not have meaning. He also argued that such an analysis does not account for the propositional nature (true/false polarity) of what the subject judges. This is stated explicitly in NL, "summary", p. 94. He also expressed it in a letter of 14 June 1913 to Russell by saying that from the proposition that S judges that aRb, must follow aRb v ~aRb, which we can construe as a hyperbolic way of saying that aRb occurs in S's judgment as a proposition (bearing implication relations). Russell at that time abandoned his early belief in propositions, and thought that propositions were no entities with which one can stand in such relations. It is evident that Wittgenstein's attack on his theory concerns Russell's conception of the nature of judgment not less than of proposition and hence includes an objection to his conception of a complex and its role in the analysis: Judging on this view is not a relation in which the subject stands to a thing (a complex), or to its various constituents.

We have briefly surveyed at least three counts on which Russell's views can hardly be seen as answering Frege's problem of complex references or as a satisfactory substitute to Frege's relegating complexity to the realm of sense: 1) his not distinguishing facts (which are propositional) from complexes (which are object-like things). 2) His un-clarity about the identity conditions of complexes and their constituents. 3) The deficiencies of his theory of judgment. These are inner problems in his positions, besides his general rejection of Frege's notion of sense. On the other hand, Frege's theory contains, as we have seen, its own problematic features: it relies on a notion of sense, to which many object; it also contains what seems to many a mysterious conception of concepts as real entities that in some magical way inherently attach to objects, and a dubious view of the

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[34] Wittgenstein attacked this in a letter to Russell of January 1913 in stating that "there cannot be different Types of things" and that "every theory of types must be rendered superfluous by a proper theory of the symbolism".

[35] On the multiple relation theory and Wittgenstein's attack on it see Griffin (1985).

[36] See McGuinness (1974).

True and the False as logical objects which are the references of propositions. In addition, it does not account for the ontological grounding of the truth of propositions. Much of Wittgenstein's *Tractatus* can be read as an effort to avoid these (and other) deficiencies in both Russell's and Frege's views.

Wittgenstein's *Tractatus Logico-Philosophicus* (TLP)

Wittgenstein, as stated above. accepted Frege's view of the primacy of language (A), but rejected Frege's notions of sense and reference. Though he used the terms *Bedeutung* and *Sinn* he restricted relevant uses of the first to names and the second to propositions, and gave them different meanings than Frege's and rejected many basic features of his theory. The most important for our concerns are: 1) the conception of the True and the False as logical objects that are the references of propositions; 2) the conception of concepts and functions as incomplete entities that are references of predicates; 3) the conception of logical constants as functors having reference and sense; 4) the lack of any account of the ontological grounding of true propositions. Wittgenstein's attack on these is often taken to be persuasive and (at least for the first two) devastating. We shall not discuss these points and Frege's possible responses, as this would amount to a separate project, but rather focus on Wittgenstein's view of complexes in the TLP and some of its problems.

The key sentence concerning complexes in TLP is 2.0201, which appears immediately after the statement "objects are simple" (2.02):

"Every statement about complexes can be resolved[37] into a statement about their constituents and into the propositions that describe the complexes completely" (2.0201).

The statement is somewhat confusing: Are there complexes (which "about complexes" in the statement suggests)? In that case, why should statements about them be resolved into others? The ending of 2.0201 is also enigmatic: What is a complete description of a complex? And why is it needed beyond the statement about the constituents? Many commentators rightly take 2.0201 to be inspired by Russell's theory of descriptions – in fact it can be seen as a sort of metaphysical generalization of it. But this problematic ending seems to have no analogue in Russell's theory, in which "The F is G" is analysed to be a statement about the concepts F and G (or their constituents), i.e. as "There is one and

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[37]   Anscomb consistently translated "zerlegen" in *NB* by " analyse"; many keep this
       translation in citing the *Tractatus*, e.g. Kenny (1975), p. 80.

only one F, and every F is G". This is arguably the statement about the constituents of which 2.0201 speaks. Where then is the additional "complete description" of the complex?

TLP 2.0201 has its origins in *Notes on Logic* of 1913 (Appendix I in NB, henceforth NL). This is a very short and interesting document. It is probably the first philosophical piece we have from Wittgenstein, very short and written at the end of the two years he was studying (mainly with Russell) at Cambridge, and it contains some of the basic ideas of TLP. With slight variations 2.0201 appears there twice. It first appears on the first page thus:

> "Every proposition which seems to be about a complex can be analysed
> into a proposition about its constituents and about the proposition which
> describes the complex perfectly; i.e. that proposition which is equivalent to
> saying the complex exists" (NL 93f).[38]

Later on it appears again thus:

> "Every statement about apparent complexes can be resolved into the
> logical sum of a statement about the constituents and a statement about the
> proposition which describes the complex completely" (NL 101b).

In some respect these formulations are more cautious than 2.0201 in speaking of what **seems** to be about a complex, or what is about **apparent** complex, instead of the T's simple but confusing "about complexes". These cautious formulations imply that in reality there are no complexes. Concerning the problematic ending of 2.0201 we have here in the opening of NL the "i.e." clause explicating it as a statement of the existence of the complex. What is the form of this statement? Is it something like Russell's analysis of descriptive statements, i.e. "there is an x such that..."? What then do the variables range over – complexes? their constituents? And how is it with complexes of other kinds (e.g. facts)?[39] Moreover, this makes the whole sentence somewhat baffling: If there are complexes, why does the beginning speak of statements that only seem to be about them?

And indeed, Wittgenstein deleted this "i.e." explanation in TLP. The reason, I surmise, is that he became more acutely opposed to the assumption of complexes and

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[38] In references to NB (and NL), a letter to the right of a number refers to paragraph.

[39] In his (1964) Black says on this i.e. clause: "what Wittgenstein has in mind here is Russell's well-known procedure for analyzing propositions containing definite descriptions" (61). This is accepted by many commentators. I suppose it is generally correct, but for the reasons stated in the text find it still unclear.

didn't want to commit himself to their existence. This, besides dispensing with the above problems, solves a systematic difficulty analogous to one solved by the theory of description: If we assume that there are complexes, their existence would be contingent, and when a complex does not exist, a statement "about it" would be meaningless. This Wittgenstein could not accept, for it would mean that the meaningfulness of a statement would depend on the truth of another statement, in contrast to e.g. 2.0211. And, just like the theory of descriptions, the solution 2.0201 suggests is that such a statement would be false, not meaningless, as explained at the head of 3.24. This then was one route of discarding complexes.

Moreover, if one takes seriously the idea that a complex "grounds" the truth of a proposition, what complex grounds the falsity of its negation? If it is another complex, as Russell (and earlier Wittgenstein in some passages in NB) appear to have assumed, we seem to have here two objects that are miraculously related as "opposites" in a way that "explains" the contradiction between a proposition and its negation (and other implications).This is most implausible. Moreover, what complex grounds the truth of a negative proposition (like "John is not at home"), or of a universal one? These and related problems preoccupied Russell and Wittgenstein at the time. They drove them to postulating "negative facts" and "universal facts", But these raised problems to which they could hardly provide satisfactory answers. The difficulty is even worse with complexes than with facts; even Russell did not speak of atomic negative complexes.[40]

And indeed the above idea of complexes as "grounding" the truth of propositions is discarded in TLP. In TLP, it is the same state of affairs (*Sachverhalt*) that corresponds to both a proposition and its negation, and is their meaning (*bedeutung*). Their difference is a difference of sense (*Sinn*) – how they relate to this state of affairs. A proposition and its negation have opposite senses. This is something that can only be shown (*zeigen*), not said (*sagen*), and it is very difficult to comprehend. This important doctrine of TLP evolved out of Wittgenstein's rejecting the Russellian (and his own previous) confounding of facts and complexes, and out of his clear understanding that facts are not (object-like) complexes. These considerations also led Wittgenstein to discard complexes as "truth-makers".

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[40]   "Atomic" because in *Theory of Knowledge* Russell speaks of "molecular complex" that "involves several propositions with such words as 'and' or 'or' or 'not' " (80). Besides problems of use and mention here it is not clear what several propositions are involved with "not".

But we are still left with state of affairs, and the very idea of state of affairs consisting of objects already raises severe difficulties. Frege, in a letter to Wittgenstein of 29 June 1918, complained that in TLP it is explained at best what is an **existing** state of affairs (that is, one that corresponds to a true proposition), not what a state of affairs is – one that corresponds to a proposition, whether true or false, which is what is needed. This, and some other of Frege's remarks there, are, I believe, of substantial philosophical significance, and deserve a fuller quoting:

> Are there also object-states (*Sachverhalten*, usually translated "states of affairs") that do not obtain? Is every combination of objects an object-state? Does it not also depend upon the means by which this combination is produced? What is that which does the combining? Can this perhaps be like gravitation with regard to the planetary system? Is this an object-state? You write: "It is essential for the thing that it can be a component of an object-state." Can now a thing be also a component of a fact? The part of the part is part of the whole. If a thing is component of a fact and every fact is part of the world, then the thing is also part of the world. ... I wish to have an example because Vesuvius is component of an object-state. Then it would seem also that the components of Vesuvius must be components of this fact; the fact would thus also consist of solidified lava. This would not seem correct to me.[41]

Each of these points raises, I believe, a serious philosophical problem whose answer in TLP is far from clear. The first, for instance, comes back to a debate Frege had with Puenjer 35 years before about existence, and to his basic position that "exist" does not mean a first-level concept, and is captured by the existential quantifier. It also pertains to Meinong's views to which Wittgenstein himself objected. The other points also raise significant and pressing problems.[42] The force of the last point, though dismissed by

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[41]   *Frege's Letters to Wittgenstein on the Tractatus*, Tr. Richard Henry Schmitt, *Bertrand Russell Society Quarterly*

[42]   To Frege's questions about the nature of the combining and what does it, have occupied people up to the present trying to find an answer in TLP (see for instance Johnston (2007).

Wittgenstein at the time, was probably appreciated by him later. Peter Geach relates in his Introduction to his edition of Frege's *Logical Investigations* that:

> Frege asked Wittgenstein if a fact was bigger than what it was a fact about; Wittgenstein told me this eventually led him to regard the Russellian view as radically confused, though at the time he thought the criticism silly.

It is not clear what "Russellian view" Wittgenstein considered as "radically confused", and whether he would also admit the force of Frege's remarks to his own view of *Sachverhalt* in TLP. Plausibly, he thought of Russell's conception of facts as object-like complexes. For, the question of whether a complex is bigger than its part is sensible, whereas whether a fact is bigger than "its part" is senseless. In a later letter of 16 September 1919 (probably after getting responses from Wittgenstein, which are lost) Frege insisted:

> You write then: "What corresponds to the elementary proposition when it is true is the obtaining of an object-state." With this you do not explain the expression "object-state", but rather the entire expression "the obtaining of an object-state" (ibid.).

I believe many commentators may have missed the point of Frege's remarks in these letters. For instance, Monk, I think, seriously underrates them in stating that they are "all concerned with terminology rather than substance"[43]. More recent literature has done better, but I believe much in the remarks is still often missed or undermined. For instance, Juliet Floyd has rightly commented on Frege's general objection expressed e.g. in his *Der Gedanke* to Wittgenstein representationalism in TLP.[44] But she does not clearly relate it to the above comments in Frege's letters and does not give full weight to their being independent of this general objection (although, possibly one of the reasons for it).[45] Many authors present Wittgenstein's notion simply by saying that a proposition "depicts reality

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[43]   Monk, R.: *Ludwig Wittgenstein*, Vintage, 1991, p. 163.

[44]   *Der Gedanke* was written before Frege received a copy of the *Tractatus* in November 1918. If Frege relates there to Wittgenstein's ideas, they are those he might have heard (but we don't know if he did) from him before the war.

[45]   Floyd, J.: "The Frege-Wittgenstein Correspondence: Interpretive Themes", part II. (http://www.bu.edu/philo/files/2011/01/Frege-WittCorrespondence.pdf)

by representing a situation", without pondering on what this might mean when there is no such situation (when the proposition is false), which is the gist of Frege's remark.[46]

There is another difficulty here: The opposition to the existence of complexes is evidently not an empirical one; it is conceptual – it amounts to the view that the notion of a complex is somewhat bogus or incoherent. If the notions of complex and simple are correlative, as many take them to be, the opposition must have its echoes with regard to the notion of simple.[47] But this would make central theses of this part of TLP problematic (cf. also 4.2211). Many commentators[48] see some aspects of the TLP theory of simples as problematic, and this one would add another. Frege and Russell would not face this problem: Frege – because in his view complexity is primarily at the realm of sense, where he has no problem in admitting its existence. Russell – although his view of simples was criticized not less than Wittgenstein's – would not face this problem, for he had no qualms about the existence of complexes as objects in the world.[49]

Whatever we think of the force of the above difficulties with the notion of state of affairs in TLP, it is clear that it is not a complex object. 3.1432 expresses the important claim that a statement is not a complex, neither is the state of affairs expressed by it. These theses are clearly against Russell's view, and possibly against some of Wittgenstein's own in NB. The proposition e.g. "2<3" should not be described as a complex (of three constituents) whose meaning is that 2<3. It should rather be described as the fact that "2" stands in a certain relation (symbolized by "<") to "3". The relation is not another component in the proposition (picture) but a feature of its structure. Likewise the state of affairs it expresses is not a complex, but propositional in its nature, i.e. that 2 is smaller than 3. This crucial claim of 3.1432 occurs already in NL 96c of 1913, where Wittgenstein

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[46] E.g. White (2006), pp. 11, 36.

[47] In many entries in NB Wittgenstein discusses various aspects of the complementarity of simple and complex. See e.g. NB 59-66 .

[48] E.g. Fogelin. See his *Wittgenstein*, Routledge, 1976, pp. 25-27.

[49] For a useful summary of Russell's views on simples see Klement, K.: "Russell's Logical Atomism" in the *Stanford Encyclopedia of Philosophy* http://plato.stanford.edu/archives/sum2015/entries/logical-atomism/

adds "Thus facts are symbolysed by facts, or more correctly: that a certain thing is the case in the symbols says that a certain thing is the case in the world".[50]

The same idea informs not only the picture theory of proposition and thought in TLP, but also the Tractarian view of perception: "To perceive a complex means to perceive that its constituents are related to one another in such and such a way" (5.5423). This means that what we perceive is propositional – it is a fact, or a state of affairs. In other words we don't perceive object-like complexes. Illustrating it with regard to the "Necker-cube" he says that in the two possible ways of seeing the figure as a cube "we really see two different facts" (*Tatsachen*, ibid.). Again he does not say that we see an object or a complex and interpret it as a fact; he says that what we see is a fact (that so and so), Complexes drop out of this account of perception, and what we remain with are just facts, which are proposition-like, not objects. This line of thought is defended and elaborated also in the Appendix to part I of *Philosophical Grammar* of 1934. Wittgenstein expands there particularly on the claim that a fact is not a complex. In connection to the problem of complexity he writes: "To say that a red circle is composed of redness and circularity, or is a complex with these component parts, is a misuse of these words and is misleading (Frege was aware of this and told me)" (200). In the remark about Frege he probably referred to the discussions they had when Wittgenstein visited Frege in Jena in December 1912, of which he wrote to Russell (26.12.12): "The complex problem is now clearer to me and I hope very much that I may solve it"[51], from which we quoted in our motto. In all these we see aspects of Wittgenstein's opposition to complexes in his conception of propositions and facts – they are definitely not object-like entities as construed by Russell.

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[50] This still escapes due attention. I must take issue e.g. with Johnston's saying that a fact in TLP is a complex of objects (n.1). This undermines the propositional nature of a fact and associates too strongly with Russell's view. The complex of 'A combined (in a certain way) with b' is not a fact. Johnston explains the combining of objects by saying "Wittgenstein by contrast will have two open hands which hold each other, close onto each other, to form the complex closed unit which is the fact." (Johnston, (2007, 244) Both this simile and its contrast (a hand covering a fist) err to this propositional character of a fact.

[51] Ludwig Wittgenstein: *Cambridge Letters*, B. McGuinness and G. H. von Wright (eds), Blackwell, Oxford, 1995.

Complexes pose also a problem from a purely ontological point of view in TLP: The world is the totality of facts (1.1). A fact is the existence of a state of affairs (2). A state of affairs is a combination of (simple) objects (2.01; cf. 3.21), and, as we saw, is not a complex. There is no place for complexes in this picture. Evidently all this presumes a complete analysis of the statements concerned. When a statement seems to be about a complex, this, as we saw, is a confusion prior to analysis, by which it is shown to be about its simple constituents. Just as Russell's "denoting concepts" of PoM and the "meanings" of descriptions of his 1904 paper disappear in the theory of descriptions of OD, and just as classes disappear in the "no class theory", and the propositions and "complex objects" of Russell's early theory of judgment disappear in his "multiple relations theory", so are the "apparent" complexes of 2.0201. They disappear under analysis and have no place in it.

This seems to me the main tenor of TLP.[52] Unfortunately, there are some formulations there that may seem to challenge it. A notable example is 3.24, in which it appears that Wittgenstein explicitly talks of the existence of complexes – what descriptions denote.[53] But this conflicts not only with what was said above but also with the fact that he obviously endorsed Russell's theory of descriptions (cf. Anscomb, 46) according to which such a description is an "incomplete symbol" that does not denote anything, and a statement comprising it is analysed so that the description "dissolves" and does not appear in the analysed form. It thus appears that 3.24 and its likes talk of only apparent situations – what only appears to be, prior to its analysis, about a complex (but in fact is not so). This is supported by Wittgenstein's saying that in such a situation there is indeterminateness in the sense of the sentence – it is not determinate what state of affairs it depicts. This is manifested in the different ways the sentence can be false: "The flower in the vase is red" can be false if there is exactly one flower in the vase and it is not red, or if

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[52]   We confine ourselves here to the complexity of "atomic propositions" and their corresponding *sachverhalten*. The complexity of molecular propositions is of a different kind, and we shall not deal with it here.

[53]   This is a prevalent tendency in much of the secondary literature. With regards the *Tractatus* Kenny for example writes: "Wittgenstein believed many of our words signify complex objects. For instance, my knife consists of a blade and a handle related in a certain way" (*Wittgenstein*, Penguin Books, 1975, p. 5) and again: "objects... can combine into complexes" (72). See also Anscomb (1971), p. 36.

there is no flower in the vase, or if there are several flowers in the vase. These ways are exposed in the Russellian analysis of the sentence, but not in its original form apparently expressing an atomic state of affairs about a complex. Hence, there is no such state of affairs.

Complexes in the Notebooks 1914-1916 (NB)

In order to better appreciate Wittgenstein's view in TLP and the difficulties he grappled with in arriving at it, it is desirable to see the differences between it and some of the thoughts in NB. NB should be taken with care, particularly when it conflicts or deviates from TLP. It often doesn't represent Wittgenstein's views, but only thoughts and considerations that occupied him at the time (and the time, let us recall, was when he served in the Austrian army during World War I). Unlike TLP and even NL it was not meant to be even shown to others. It is notable that some important theses of TLP appear already in NL, but are somehow blurred or discarded in NB, and then come back to the fore in TLP. Complexes is a case in point. In NB Wittgenstein speaks extensively of complexes and complex objects, which names denote (e.g. 47k; 50b, h; 52m) and grapples with some of the problems it raises. It is a major topic there.[54] There are some relics of it in TLP, though, as stated above, I believe that most of these theses are abandoned or rejected in TLP, and the main tenor of TLP is to do away with complexes.[55] In TLP a proposition is a picture of a state of affairs – this is its sense – and a state of affairs in TLP is not a complex – an object-like thing – but is essentially propositional (3.1432).

Like Russell, Wittgenstein took seriously the intuition that if a proposition P is true, there is something in the world to which P corresponds, and which "makes" it true. This thing, Wittgenstein concluded in some passages in NB, is not a fact, for a fact is not a thing, hence he then thought, it is a complex and it is designated by a complex sign (48c). This is a deep-rooted intuition about truth, which informs many representational theories of meaning and correspondence theories of truth. And, as explained above, it is something Wittgenstein may have found missing in Frege. For, to this idea of something corresponding to a true thought, as noted above, one can hardly find an analogue in Frege.

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[54]   See especially the notes from 14 June 1915 to the end of the year.

[55]   I am thus happy to agree with Potter's remark: "The fundamental idea of his eliminative programme is that no proposition is genuinely about a complex: the apparent complex will always disappear on analysis" (44).

It is still alive in TLP, but there facts or existing states of affairs, not complexes, serve as the ontological correspondents of true propositions.

A peculiar feature of many entries in NB, in contrast, is that they assimilate a proposition to a description. This is why Wittgenstein could speak there of a proposition as a picture of a complex (70d, 52n). He speaks of a proposition as describing a complex, and this seems to blur the distinction between naming or describing and stating or asserting: "A proposition describes a complex in the kind of way that Russellian descriptions do ... by means of its logical properties" (16b). A complex in much of NB is an object, or a thing. He says that one can point to a complex (watch lying on the table) and say "this is what I mean" (70d). Incidentally, Wittgenstein reverses in this connection Russell's notions of knowledge by description and by acquaintance, and says that the former applies to simple objects, the latter to complexes (50h), whereas for Russell we know by acquaintance the constituents of a proposition we understand; the denotations (which are often complexes) – by description.[56] Whether facts are also such complexes is unclear in NB. He seems, on the one hand, to hold that propositions are not names, but to regard propositions as descriptions of complexes, on the other. Proposition and fact are somehow assimilated there to description and complex in a way that was forcefully rejected in TLP.[57]

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Great philosophers often differ on fundamentals in a way that makes disputes on particular issues very difficult to present adequately. The issue of complexes in Frege Russell and Wittgenstein is a case in point. It touches on the nature of truth and logic, language and the fundamentals of a theory of meaning. Our discussion was therefore inevitably very partial. In one sentence we can conclude it by the following: Frege's theory of reference does not allow for complexity, which is therefore relegated primarily to the realm of sense; Russell, repudiating Frege's notion of sense, saw complexes, facts included, as structured objects that are directly accessible to our cognition; Wittgenstein, insisting on the primacy of language and on the fact/object distinction, discarded complexes in favor of facts.

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[56]   These notions and their relationships are discussed in Bar-Elli (1989).

[57]   On complexes, simples and definiteness of sense see an extensive discussion in
        NB 63-70 (June 1915).

Less misleadingly and in more detail: The problematic nature of complexes was set up in Frege's theory of reference. For Frege things in the world are given to us as references of expressions in logically analyzed propositions (Principle A). Since in general the reference of a part of an expression is no part of the reference of the whole (principle B), things, conceived as references, are not complex. Complexity, structure constituents, and analysis belong primarily to the realm of sense (C). Russell rejected Frege's notion of sense. His reasons in the appendix to PoM and in OD are notoriously difficult to follow and hardly persuasive. But deeply, as regards complexes and analysis, there is another fundamental reason for his rejection of (C): it is his rejection of (A) – the Fregean conviction that things in the world are given to us as references of (logically analyzed) linguistic expressions, i.e. via our theory of meaning. However, he was wavering and indecisive about the exact meaning of some of his basic notions, like proposition, variable, complex, and fact.[58] But he thought that we have direct cognitive access to their nature, their structure and constituents, independently of the particulars of a theory of meaning. Wittgenstein shared the above Fregean conviction in the primacy of language (A), as well as some other basic Fregean theses, but objected to many others, some of which were mentioned above. Much of this was also at the basis of his disputes with Russell. The result, for our concerns, was a rejection of complexes, and hence – of non-propositional analysis. But he sided with Russell (against Frege) in maintaining that when a proposition (or judgment) is true, there is something in the world that "makes" it true. In TLP facts (existing states of affairs) fulfill that role, and facts, he emphasized against Russell, are not complex objects, cannot be named, described or referred to (exceptional special cases aside), but are essentially propositional and can only be depicted by other propositional facts. The exact nature of these propositional facts, severely challenged by Frege, and their accessibility to our cognition independently of language, as proclaimed by Russell, are fundamental issues that make the above controversies philosophically alive.

Gilead Bar-Elli, Jerusalem, July 2015

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[58]   Frege was one of the first to point this out with regards to PM, see his letter to P. Jourdain of 28 January 1914, PMC 81-84.

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